Demonstration Tests – Attribute Sample Size

One common question for demonstration tests is determining the sample size to meet certain conditions. This article explores the case where the test results are either conforming or nonconforming.

The assumptions for the statistical method are:

- Each item in the sample is subject to inspection or test
- Each inspection or test result is either pass or fail
- The inspections or tests are independent
- The probability of pass is specified

The assumptions lead to a series of Bernoulli trials. In each trial, there are only two possible outcomes and results of each trial in independent of the result form any other trial. A common example is flipping a fair coin. The coin doesn't have a memory, so the result of a flip doesn't depend on the result of prior flips. There are only two possible outcomes – heads or tails.

The binomial distribution models a series of Bernoulli trials. In this case, each Bernoulli trial outcome is either conforming or nonconforming.

The process starts by setting three parameters:

P_0, the probability that the number of failures doesn't exceed the specified number
p, the probability that any individual item fails the inspection or test
f, the maximum number of failures allowed

By convention, the analysis uses the following values. However, other values may apply.

$P_0 = 0.05$ (the probability of passes is 0.95)
$p = 0.01$ (the probability of a single item passing is 0.99)
$f = 0$

The underlying statistical distribution is the binomial described by the following equation.

$$P_0 = P(F \leq f) = \sum_{i=0}^{f} \binom{n}{i} p^{n-i} (1-p)^i$$

where F is the number of observed failures

For example, consider a sample of size 13, $n = 13$, from a process with a nonconformance rate of 0.05, $p = 0.05$. We are interested in the probability that the sample has 2 or fewer nonconforming items, $F \leq 2$. Notice that this is "cumulative", so we are interested in the probability of 0 nonconforming items, 1 nonconforming item, or 2 nonconforming items. Notice that the summation is over i and adds each of the three cases: $i=0$, $i= 1$, and $i = 2$.

The Excel function BINOM.DIST(number_s, trials ,probability_s, cumulative) can perform the calculation. Note that is this case we are discussing nonconforming items, so success means a nonconforming item.

The BINOM.DIST function syntax has the following arguments:

number_s   The number of successes in trials

trials   The number of independent trials

probability_s   The probability of success on each trial

cumulative   A logical value that determines the form of the function. If cumulative is TRUE, then BINOM.DIST returns the cumulative distribution function, if FALSE, it returns the probability mass function

For our example the function is BINOM.DIST(2, 13, 0.05, TRUE) which returns the value 0.975.

The probability of getting 0, 1, or 2 nonconforming items from a sample of 13 is about 97.5%, when the process produces 5% nonconforming items.

Sample Size

The process starts with the value for *p*. The pick a value for *n* and for *f* and determine if they meet P_0. If not, continue to select values for *n* and *f* until the result meets the specified P_0. There will be multiple combinations that satisfy the requirement. Note that, because *n* and *f* must be an integer, the result may not be the exact value of P_0.

Special Case, *f = 0*

When f = 0, the binomial distribution formula reduces to

$$P_0 = \binom{n}{0} p^{n-0}(1-p)^0 = p^n$$

In this case it is easy to solve for *n* by taking the logarithm of each side.

$$ln(P_0) = ln(p^n)$$

$$n = \frac{ln(P_0)}{ln(p)}$$

In this case, the question often has a different formulation. Instead of the nonconformance rate, *p*, the question uses the conformance rate, often termed reliability, $R = 1 - p$. Then the confidence becomes $C = 1 - P_0$.

The solution to the sample size then becomes

$$n = \frac{ln(1-C)}{ln(R)}$$

Example: What is the sample size to be for 95% confidence with zeros failures and a reliability of 99%?

$$n = \frac{ln(1 - 0.95)}{ln(0.99)} = 298.07$$

In practice, round to the next highest integer or 299.